**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans -We have a normal distribution with µ= 45 and *σ* = 8.0.

Let X be the amount of time it takes to complete the repair on a customer's car.

To finish in one hour you must have X ≤ 50 so the question is to find Pr(X > 50).

Pr(X > 50) = 1 - Pr(X ≤ 50).

Z = (X - µ)/ = (X - 45)/8.0

Pr(X ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or **0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans -

normal distribution *μ* = 38 and *σ* = 6.

Let X be the number of employees.

a) Probability of employees greater than age of Pr(X>44)

Pr(X > 44) = 1 - Pr(X ≤ 44).

Z = (X -*μ*)/ = (X - 38)/6

Pr(X ≤ 44) = Pr(Z ≤ (44 - 38)/6) = Pr(Z ≤ 1)=84.1345%

Probability that the employee will be greater than age of 44 =

100-84.1345=15.86%

So, the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-0.5= 34.1345%

the statement that “More employees at the processing center are older than 44 than between 38 and 44” is **TRUE**.

b) Probability of employees less than age of Pr(X<30).

Z = (X -*μ*)/ = (30 - 38)/6

Pr(X ≤ 30) = Pr(Z ≤ (30 - 38)/6) = Pr(Z ≤ -1.333)=9.12%

So, the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48 (36 employees).

the statement B of the question is also **TRUE**.

1. If *X1* ~ *N* (μ, σ2) and *X*2 ~ *N* (μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans - if X ∼ N (μ1, σ1^2), and Y ∼ N (μ2, σ2^2) are two independent random variables then X + Y ∼ N (μ1 + μ2, σ1^2 + σ2^2), and X − Y ∼ N (μ1 − μ2, σ1^2 + σ2^2).

if Z = aX + bY , where X and Y are as defined above,

i.e Z is linear combination of X and Y

then Z ∼ N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2 ).

2X1~ N (2 u,4 σ^2) and

X1+X2 ~ N (μ + μ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

2X1-(X1+X2) = N (4μ,6 σ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: - To find the values of a and b, we need to find the range of values within which the middle 99% of the data lie. Using the properties of the normal distribution, we know that 99% of the data lies within 2.58 standard deviations of the mean. Therefore, we can use the following formula to find the values of a and b:

a = mean + 2.58 \* standard deviation

b = mean - 2.58 \* standard deviation

Substituting the values from the question, we get:

a = 100 + 2.58 \* √20 ≈ 109.9

b = 100 - 2.58 \* √20 ≈ 90.1

Thus, the values of a and b are 109.9 and 90.1, respectively. These values are symmetric about the mean (100), and they represent the range of values within which the middle 99% of the data lie. Therefore, the correct answer is

* E. 90.1, 109.9.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans - To find the Rupee range that contains 95% probability for the annual profit of the company, we need to first find the distribution of the total profit, which is the sum of the profits of the two divisions. Since the profits of the two divisions are independent, the total profit has a normal distribution with mean μ = 5+7 = 12 and variance σ^2 = 3^2 + 4^2 = 25.

To convert the profit to Rupees, we need to multiply by the conversion factor of Rs. 45 per dollar. Therefore, the total profit in Rupees has a normal distribution with mean μ = 12 \* 45 = 540 and variance σ^2 = 25 \* 45^2 = 50625.

We can now use the properties of the normal distribution to find the Rupee range that contains 95% probability for the annual profit of the company. Since the distribution is symmetric, we can find the range by calculating the z-scores corresponding to the 2.5th and 97.5th percentiles of the standard normal distribution. These z-scores can be found using a table or calculator and are approximately -1.96 and 1.96, respectively.

Using the formula for standardizing a normal distribution, we can find the corresponding values of the total profit in Rupees:

Lower bound = μ + zσ = 540 - 1.96 \* √50625 ≈ 438.3 million Rupees

Upper bound = μ + zσ = 540 + 1.96 \* √50625 ≈ 641.7 million Rupees

Therefore, the Rupee range that contains 95% probability for the annual profit of the company is approximately 438.3 million to 641.7 million Rupees, centered on the mean of 540 million Rupees.

B. To find the 5th percentile of profit in Rupees for the company, we need to first find the 5th percentile of the standard normal distribution, which is approximately -1.645. We can then use the formula for standardizing a normal distribution to find the corresponding value of the total profit in Rupees:

Total profit in Rupees = μ + zσ = 540 + (-1.645) \* √50625 ≈ 425.4 million Rupees

Therefore, the 5th percentile of profit in Rupees for the company is approximately 425.4 million Rupees.

C. To determine which of the two divisions has a larger probability of making a loss in a given year, we need to calculate the probability of each division making a loss. A division makes a loss if its profit is negative.

For Profit1, the probability of making a loss is the probability that a standard normal variable with mean 5 and standard deviation 3 is less than zero. This can be calculated using a standard normal distribution table or calculator and is approximately 0.1587.

For Profit2, the probability of making a loss is the probability that a standard normal variable with mean 7 and standard deviation 4 is less than zero. This can also be calculated using a standard normal distribution table or calculator and is approximately 0.0668.

Therefore, Profit1 has a larger probability of making a loss in a given year.